NOTATION

c, heat capacity; D, diameter; F, area of the transverse cross section; G, flow rate; Gr_A, Grashof number; g, acceleration of gravity; h, enthalpy; p, pressure; Π, perimeter; Re, Reynolds number; s, complex Laplace transform variable; T, temperature; t, time; x, distance along the channel; λ, thermal conductivity; ω, circular frequency of the pulsations; and ρ, specific density. The indices denote the following: in, at the inlet to the channel; out, at the outlet from the channel; liq, the flow; 0, in front of the throttle at the channel input; m, the pseudocritical temperature; N, in front of the throttle at the channel output; w and c, the pipe wall.

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UNIVERSAL PROFILES AND LAW OF TURBULENT NEAR-WALL HEAT AND MASS TRANSFER

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Universal distributions that do not contain empirical constants are obtained in the turbulent core of the mean longitudinal velocity, temperature and concentration of a substance for arbitrary Prandtl and Schmidt molecular numbers.

The development of modern engineering in the domain that is characterized by the presence of internal or external heat and mass transfer on streamlined surfaces is greatly retarded because there is no single description of this phenomenon for different values of the Pradtl and Schmidt numbers based on universal distributions of the velocity, temperature, and concentration of a substance that do not contain empirical constants, and a law of turbulent heat and mass transfer. Precisely the absence of experimental coefficients in such generalized relationships permits their effective application in computations of complex flow conditions characteristic for energy-saving aggregates, consequently, setting up universal dependences is of great scientific and practical interest. An attempt is made in this paper to obtain such generalized relationships and the possibility is shown of their utilization to describe specific flows.

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Applying the principle of superposition of molecular and turbulent viscosity and the conception of a viscous sublayer with different thicknesses for the hydrodynamic and thermal boundary layers, M. D. Millionshchikov [1] obtained general dependences of the heat and mass transfer on the Prandtl and Schmidt numbers by including both small, substantially less than one, and very much larger values than one. In this case

$$\vartheta^+ = A\ln\left[1+\frac{\Pr}{A}(y^+-\delta_{0T}^+)\right]+\Pr\delta_{0T}^+, \tag{1}$$

is valid for the temperature profile in a turbulent core, as is, correspondingly, the Stanton number

$$\mathrm{St}=\frac{1}{u_\delta^+\vartheta_{\delta_T}^+}=\left[u_\delta^+\left(A\ln\left[1+\frac{\Pr}{A}(\delta^+-\delta_{0T}^+)\right]+\Pr\delta_{0T}^+\right)\right]^{-1}. \tag{2}$$

Here the empirical constant is $A = 2.57$. For $\Pr \geq 1$ $\delta_{0T} = \delta_0 \Pr^{-0.25}$ and for $\Pr < 1$ $\delta_{0T} = \delta_0$, where δ_0 and δ_{0T} are the thicknesses of the hydrodynamic and thermal viscous sublayers.

The temperature distribution (1) and the law of turbulent heat and mass transfer (2) (and its modification for the mean values of the velocity $\bar{u}$ and temperature $\bar{v}$ over the stream) are obtained without constructing interpolation formulas with additional test coefficients, and are of general nature and satisfactorily describe the known experimental data concerning simple flow types on a plate, in a pipe and a plane channel. If several perturbing factors such as a mass force field, nonisothermy, mass transfer on boundary surfaces, etc. act separately or simultaneously on the stream, then because of the empirical constant A the relationships (1) and (2) cannot be applied.

Another more widespread viewpoint exists also [2, 3] according to which logarithmic temperature distributions hold in a turbulent boundary layer on the basis of similarity theory

$$\vartheta^+ = A\ln y^+ + B(\Pr) \tag{3}$$

and

$$\vartheta_{\delta_T}^+ - \vartheta^+ = -A\ln(y/\delta_T) + B_1, \tag{4}$$

i.e.

$$\mathrm{St} = [u_\delta^+(A\ln\delta_T^+ + B(\Pr) + B_1)]^{-1}. \tag{5}$$

Here A, B_1 are empirical constants, B is an empirical function dependent on the molecular Prandtly number Pr. The equation (3) refers to a zone located directly behind the transition domain, and (4) to the outer part of the boundary layer.

The principal distinction between (1) and (3) is the absence of the empirical function B(Pr). For $\Pr(y^+ - \delta_{0T}^+)/A \gg 1$

$$\vartheta^+ = A\ln(y^+-\delta_{0T}^+) + \Pr\delta_{0T}^+ + A\ln\frac{\Pr}{A} \tag{6}$$

from (1), which corresponds to (3) for $y^+ \gg \delta_{0T}^+$.

Since (1), (2) and (3)-(5) are obtained, despite their differences, from authentic assumptions at a given level of development of the theory of the turbulent boundary layer, then it is conceivable that more general relationships can exist that do not contain the empirical constants, for which (1)-(5) are particular cases.

Generalized profiles of the mean longitudinal velocity, temperature, and concentration of substance in a turbulent core [5] were obtained in a turbulent boundary layer model [4-7] representing a further development of the fundamental ideas of M. D. Millionshchikov [8, 9]

$$\frac{u_i-u_{0i}}{u_{\delta_i}-u_{0i}}=\frac{\ln(y/\delta_{0i})}{\ln(\delta_i/\delta_{0i})}, \tag{7}$$

where $u_1 = u$, $u_2 = \vartheta$, $u_3 = c$, $\delta_{01} = \delta_0$, $\delta_{02} = \delta_{0T}$, $\delta_{03} = \delta_{0C}$, $\delta_1 = \delta$, $\delta_2 = \delta_T$, $\delta_3 = \delta_C$. The subscripts i = 1, 2, 3 refer, respectively, to momentum, heat, and substance transfer.

For $\Pr - \mathrm{Sc} = 1$ [4, 6]

$$\frac{u_i^+ - 1}{u_{\delta_i}^+ - 1} = \frac{\ln y^+}{\ln \delta_i^+}. \tag{8}$$

Here $u_i^+ = u_i/u_{*i}$, $u_{*1} = u_*$, $u_{*2} = \vartheta_*$, $u_{*3} = c_*$.

The expressions (7) and (8) act in the domain of the turbulent core, which corresponds approximately to a segment measured from the outer boundary to the Karman transition domain equal to 90-95% of the boundary layer thickness. Formula (7) is independent of the replacement of the scale δ_{0i} by any other belonging to the domain $[\delta_{0i}, \delta_i]$. In this sense, such scales are equivalent [7]. The relationship (8) can be obtained from the assumption that the viscous scale $\ell_* = \nu/u_*$ can also be equivalent to the scale for (7). Then according to [7], upon replacement of δ_{0i} by ℓ_* in (7), the expression (8) is obtained directly which describes specific profiles, as does (7), in a turbulent core, for instance in the segment $[\delta_{0i}{}^+, \delta_i{}^+]$ when the Karman transition domain is neglected.

Known logarithmic representations of the mean temperature (3) and (4) satisfy (7) since upon substitution of ϑ^+, $\vartheta_{0T}{}^+$, and $\vartheta_{\delta T}{}^+$ determined from (3) or (4) into the left side of (7) an equation identical to (7) is obtained for the temperature profile

$$\frac{\vartheta^+ - \vartheta_{0T}^+}{\vartheta_{\delta_T}^+ - \vartheta_{0T}^+} = \frac{\ln (y/\delta_{0T})}{\ln (\delta_T/\delta_{0T})}. \tag{9}$$

At first glance, the independence of the general expression (7) from the molecular Prandtl Pr and Schmidt Sc numbers is justified since the empirical function B(Pr) in (3) that takes account of the temperature profile deformation because of a change in the number Pr was eliminated from the consideration upon going from (3) over to (9).

In turn, utilization of (1) for substitution in (7) yields another result

$$\frac{\vartheta^+ - \vartheta_{0T}^+}{\vartheta_{\delta_T}^+ - \vartheta_{0T}^+} = \frac{\ln \left[1 + \dfrac{\Pr}{A}(y^+ - \delta_{0T}^+)\right]}{\ln \left[1 + \dfrac{\Pr}{A}(\delta_T^+ - \delta_{0T}^+)\right]}, \tag{10}$$

where $\vartheta_{0T}{}^+ = \Pr\delta_{0T}{}^+$ according to [1].

The question arises of the applicability of (9) and (10). The presence of the experimental coefficient A in (10) constrains the possibility of utilization of this equation but because of the presence of the Prandtl number its informability is expanded as compared to (9). In this sense both relationships are not fundamental.

From general considerations universal expressions for the profiles of the velocity, temperature, and concentration of a substance should not contain empirical constants but they include the criterial number ν/ν_i to take account of the specifics of a portable substance. Here $\nu_1 = \nu$, $\nu_2 = a$, $\nu_3 = D$, i.e., $\nu/\nu_1 = 1$, $\nu/\nu_2 = \nu/a = \Pr$, $\nu/\nu_3 = \nu/D = \mathrm{Sc}$. Then on the basis of the model [4-7]

$$\frac{u_i - u_{0i}}{u_{\delta_i} - u_{0i}} = \frac{\ln f\left(\dfrac{\nu}{\nu_i}, y, \delta_{0i}\right)}{\ln f\left(\dfrac{\nu}{\nu_i}, \delta_i, \delta_{0i}\right)} \tag{11}$$

by analogy with (7) and (10). Here, since $\nu/\nu_1 = \nu/\nu = 1$ for the velocity profile and $\delta_{0i} = \ell_* \equiv \nu/u_*$ for a zero pressure gradient, the expression (11) should go over into (7) and then into (8). For the profiles of the temperature and the concentration of substance, (11) should also be transformed into (7) and (8) for $\nu/\nu_2 = \Pr = 1$ and $\nu/\nu_3 = \mathrm{Sc} = 1$.

The function f in (11) should take account of the characteristics features of the function f in (9) and (10) since they are particular cases of (11). Measurement of the distance y on the basis of (10) is made from the boundaries δ_{0i} of the viscous sublayers, i.e., the argument is in the form of the difference $y - \delta_{0i}$. Since it should also be dimensionless, in the general case according to (9) it takes the form of the ratio $(y - \delta_{0i})/\delta_{0i}$. Taking account of the criterial numbers ν/ν_i as in (10) can be realized by multiplying the argument $(y - \delta_{0i})/\delta_{0i}$ by ν/ν_i. Therefore, the function f in a first approximation will have the form

$$f = \frac{\nu}{\nu_i}\left(\frac{y - \delta_{0i}}{\delta_{0i}}\right) + 1,$$

where analogously to (10) the component equal to one is selected from the condition corresponding to the fact that $\vartheta_i^+ = \vartheta_{0i}^+$ for $y = \delta_{0i}$ and, respectively, $\ln f(y = \delta_{0i}) = 0$.

Then the generalized profiles of the mean longitudinal velocity, temperature and concentration of a substance in a turbulent core will have a single form independently of the values of the Prandtl Pr and Schmidt Sc numbers:

$$\frac{u_i - u_{0i}}{u_{\delta_i} - u_{0i}} \equiv U_i = R_i \equiv \frac{\ln\left[\frac{\nu}{\nu_i}\left(\frac{y - \delta_{0i}}{\delta_{0i}}\right) + 1\right]}{\ln\left[\frac{\nu}{\nu_i}\left(\frac{\delta_i - \delta_{0i})}{\delta_{0i}}\right) + 1\right]}. \tag{12}$$

The distribution of the mean longitudinal velocity (the subscript i = 1) from (12) is derived by substituting $\nu_1 = \nu$, $u_1 = u$, then

$$\frac{u - u_0}{u_\delta - u_0} \equiv U = R \equiv \frac{\ln(y/\delta_0)}{\ln(\delta/\delta_0)}, \tag{13}$$

which is identical to (7), and then δ_0 is replaced by ℓ_* for a zero pressure gradient:

$$\frac{u^+ - 1}{u_\delta^+ - 1} \equiv \tilde{U} = \tilde{R} \equiv \frac{\ln y^+}{\ln \delta^+}, \tag{14}$$

which agrees with (8).

The mean temperature distribution (subscript i = 2) can be obtained from (12) by taking into account that $\nu/\nu_2 = \nu/a = \mathrm{Pr}$, $u_2 = \vartheta$, $\delta_2 = \delta_T$, $\delta_{02} = \delta_{0T}$:

$$\frac{\vartheta - \vartheta_{0T}}{\vartheta_{\delta_T} - \vartheta_{0T}} \equiv \Theta = R_\Theta \equiv \frac{\ln\left[\mathrm{Pr}\left(\frac{y - \delta_{0T}}{\delta_{0T}}\right) + 1\right]}{\ln\left[\mathrm{Pr}\left(\frac{\delta_T - \delta_{0T}}{\delta_{0T}}\right) + 1\right]}. \tag{15}$$

For Pr = 1, (9) follows directly from (15).

According to [1], $\delta_{0T} = \delta_0$ for $\mathrm{Pr} \le 1$, i.e., the thickness of the thermal viscous sublayer equals the thickness of the hydrodynamic viscous sublayer. Taking into account that the viscous scale $\ell_* = \nu/u_*$ can be equivalent to the scale δ_0, which would hold when going from (7) to (8) when the functions δ_{0i} would be replaced by ℓ_*,

$$\frac{\vartheta^+ - \vartheta^+(y^+ = 1)}{\vartheta_{\delta_T}^+ - \vartheta^+(y^+ = 1)} = \frac{\ln[\mathrm{Pr}(y^+ - 1) + 1]}{\ln[\mathrm{Pr}(\delta_T^+ - 1) + 1]} \tag{16}$$

can be obtained analogously from (15).

Since

$$\vartheta^+ = \mathrm{Pr}\, y^+ \tag{17}$$

for molecular transfer near the wall where the Fourier law operates [2], then from (16)

$$\frac{\vartheta^+ - \mathrm{Pr}}{\vartheta_{\delta_T}^+ - \mathrm{Pr}} \equiv \tilde{\Theta} = \tilde{R}_\Theta \equiv \frac{\ln[\mathrm{Pr}(y^+ - 1) + 1]}{\ln[\mathrm{Pr}(\delta_T^+ - 1) + 1]}. \tag{18}$$

Formula (18) is the universal temperature profile in a turbulent core for $\Pr \leq 1$. As $\Pr \to 1$ (18) goes over into (8):

$$\frac{\vartheta^+ - 1}{\vartheta^+_{\delta_T} - 1} \equiv \tilde{\Theta} = \tilde{R}_\Theta \equiv \frac{\ln y^+}{\ln \delta^+_T}. \tag{19}$$

Separating the functions u_i, y running over the section in (12) from the boundary u_{δ_i}, δ_i on the main flow side, we can obtain

$$\frac{\ln\left[\frac{\nu}{\nu_i}\left(\frac{y-\delta_{0i}}{\delta_{0i}}\right)+1\right]}{u_i^+ - u_{0i}^+} \equiv \Psi_i(x,\ y) = \Psi_{\delta_i}(x) \equiv \frac{\ln\left[\frac{\nu}{\nu_i}\left(\frac{\delta_i-\delta_{0i}}{\delta_{0i}}\right)+1\right]}{u^+_{\delta_i} - u^+_{0i}}, \tag{20}$$

where $\Psi_1 = \Psi$, $\Psi_2 = \chi$, $\Psi_3 = \Omega$ are criterial functions of the turbulent core that remain constant within it and equal to eigenvalues on the outer boundary for $y = \delta_i$.

From (20) for the momentum transfer

$$\frac{\ln(y/\delta_0)}{u^+ - u_0^+} \equiv \Psi(x,\ y) = \Psi_\delta(x) \equiv \frac{\ln(\delta/\delta_0)}{u^+_\delta - u^+_0}, \tag{21}$$

for zero pressure gradient

$$\frac{\ln y^+}{u^+ - 1} \equiv \tilde{\Psi}(x,\ y) = \tilde{\Psi}_\delta(x) \equiv \frac{\ln \delta^+}{u^+_\delta - 1}, \tag{22}$$

for heat transport for arbitrary values of Pr

$$\frac{\ln\left[\Pr\left(\frac{y-\delta_{0T}}{\delta_{0T}}\right)+1\right]}{\vartheta^+ - \vartheta^+_{0T}} \equiv \chi(x,\ y) = \chi_{\delta_T}(x) \equiv \frac{\ln\left[\Pr\left(\frac{\delta_T-\delta_{0T}}{\delta_{0T}}\right)+1\right]}{\vartheta^+_{\delta_T} - \vartheta^+_{0T}}, \tag{23}$$

for $\Pr \leq 1$

$$\frac{\ln[\Pr(y^+ - 1) + 1]}{\vartheta^+ - \Pr} \equiv \tilde{\chi}(x,\ y) = \tilde{\chi}_{\delta_T}(x) \equiv \frac{\ln[\Pr(\delta^+_T - 1) + 1]}{\vartheta^+_{\delta_T} - \Pr} \tag{24}$$

for substance transport for any Sc

$$\frac{\ln\left[\mathrm{Sc}\left(\frac{y-\delta_{0c}}{\delta_{0c}}\right)+1\right]}{c^+ - c^+_{0c}} \equiv \Omega(x,\ y) = \Omega_{\delta_c}(x) \equiv \frac{\ln\left[\mathrm{Sc}\left(\frac{\delta_c-\delta_{0c}}{\delta_{0c}}\right)+1\right]}{c^+_{\delta_c} - c^+_{0c}}, \tag{25}$$

for $\mathrm{Sc} \leq 1$

$$\frac{\ln[\mathrm{Sc}(y^+ - 1) + 1]}{c^+ - \mathrm{Sc}} \equiv \tilde{\Omega}(x,\ y) = \tilde{\Omega}_{\delta_c}(x) \equiv \frac{\ln[\mathrm{Sc}(\delta^+_c - 1) + 1]}{c^+_{\delta_c} - \mathrm{Sc}}. \tag{26}$$

Expanding the running criterial functions $\Psi_i(x,\ y)$ in (21)-(26), logarithmic distributions of the velocity, temperature, and concentration of substance can be obtained in contrast to the formulas (1), (3), (4) of similar form but not containing empirical constants. The temperature profiles transformed from (23) and (24) have the form

$$\vartheta^+ = \frac{1}{\chi_{\delta_T}} \ln\left[\Pr\left(\frac{y-\delta_{0T}}{\delta_{0T}}\right)+1\right] + \vartheta^+_{0T}, \tag{27}$$

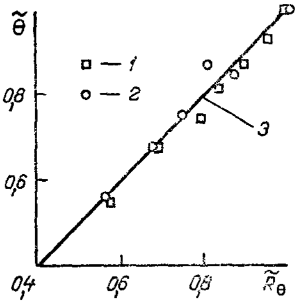

Fig. 1

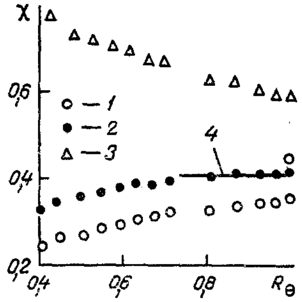

Fig. 2

Fig. 1. Temperature profiles $\tilde{\Theta}(\tilde{R}_\theta)$ for mercury flow (Pr = 0.026) in a circular tube from the data presented in [10]: 1) Re = $110\cdot10^3$, $\tilde{\chi}_{\delta_T} = 0.41$; 2) $130\cdot10^3$ and 0.45; 3) computation using (18).

Fig. 2. The dependence of $\chi(R_\theta)$ on the selection of δ_{0T}^+ and ϑ_{0T}^+ according to the data of [12] for water flow in a plane channel for $\mathrm{Pr}_w = 6.14$: 1) $\delta_{0T}^+ = 7.5$, $\vartheta_{0T}^+ = 46.0$, $\chi_{\delta_T} = 0.35$; 2) 8.0; 49.12 and 0.41; 3) 9.2; 53.9 and 0.59; 4) computation using (23).

for Pr ≤ 1

$$\vartheta^+ = \frac{1}{\tilde{\chi}_{\delta_T}} \ln[\mathrm{Pr}(y^+ - 1) + 1] + \mathrm{Pr}. \tag{28}$$

On the basis of [6] the analogy between turbulent momentum, heat, and substance transport will have the form

$$\mathrm{Pr}_t = \frac{\Psi_\delta}{\chi_{\delta_T}}, \tag{29}$$

$$\mathrm{Sc}_t = \frac{\Psi_\delta}{\Omega_{\delta_c}}. \tag{30}$$

For a zero pressure gradient in (29) and (30) $\Psi_\delta = \tilde{\Psi}_\delta$, for Pr ≤ 1 $\chi_{\delta_T} = \tilde{\chi}_{\delta_T}$ and for Sc ≤ 1 $\Omega_{\delta_c} = \tilde{\Omega}_{\delta_c}$.

The law of turbulent near-wall heat transport should set up a connection between the temperature drop $\vartheta_{\delta_T} = T_w - T_{\delta_T}$ and the characteristic temperature $\vartheta_* = q_w/\rho c_p u_*$. From (21), (29), and (23)

$$\vartheta_{\delta_T}^+ \equiv \frac{T_w - T_{\delta_T}}{\vartheta_*} = \frac{\mathrm{Pr}_t(u_\delta^+ - u_0^+)\ln\left[\mathrm{Pr}\left(\frac{\delta_T - \delta_{0T}}{\delta_{0T}}\right) + 1\right]}{\ln(\delta/\delta_0)} + \vartheta_{0T}^+. \tag{31}$$

Correspondingly, the law of near-wall mass transfer that displays the connection between the difference of the relative mass concentrations of passive impurity $c_{\delta_c} = m_w - m_{\delta_c}$ and the characteristic concentration $c_* = j_w/\rho u_*$ has the following form on the basis of (21), (30), and (25)

$$c_{\delta_c}^+ \equiv \frac{m_w - m_{\delta_c}}{c_*} = \frac{\mathrm{Sc}_t(u_\delta^+ - u_0^+)\ln\left[\mathrm{Sc}\left(\frac{\delta_c - \delta_{0c}}{\delta_{0c}}\right) + 1\right]}{\ln(\delta/\delta_0)} + c_{0c}^+. \tag{32}$$

The law (31), (32) can also be represented traditionally in the form of appropriate expressions for the criterial Stanton numbers $\mathrm{St} = 1/u_\delta^+\vartheta_{\delta_T}^+$ and $\mathrm{St}_c = 1/u_\delta^+ c_{\delta_c}^+$.

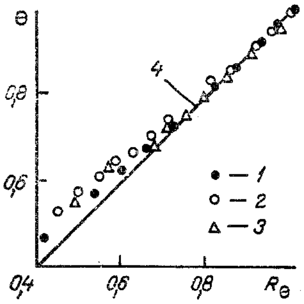

Fig. 3. Temperature profiles $\Theta(R_\theta)$ for water flow in a plane channel according to data from [12]: 1) $Pr_w = 6.33$, $\chi_{\delta T} = 0.43$, $\delta_{0T}^+ = 7.5$; 2) 6.14; 0.41 and 8.0; 3) 6.76; 0.41 and 7.0; 4) computation using (15).

The temperature distribution (18) for $Pr \le 1$ is compared in Fig. 1 with the experimental data represented in [10] and concerning the flow of mercury in a circular tube ($Pr = 0.026$). The domain $\tilde{R}_\Theta$ [0.7; 1.0] corresponds to the turbulent core. The satisfactory correlation is seen between the universal distribution (18) not containing empirical constants and the data presented. Values of the criterial function $\tilde{\chi}_{\delta T}$ for $P \ll 1$ here correspond to the values of $\tilde{\chi}_{\delta T}$ for $Pr = 1$ mentioned in [6].

Since the dynamic criterial function Ψ_δ in (29) is independent of Pr while the values of the thermal criterial function $\chi_{\delta T}$ remain practically constant for $Pr \le 1$, then Pr_t in (29) is also independent of the molecular Prandtl number or is a weak function of it. Analogously it can be considered for $Pr > 1$ that $\chi_{\delta T}$ equal the values $Pr \le 1$ under identical flow conditions, which substantially simplifies the temperature profile computation when using the relationships (15).

The principal distinction between (15) which is true for arbitrary values of Pr and its particular case (18) corresponding to $Pr \le 1$ is the presence of the boundary function δ_{0T} that is physically the thickness of the thermal viscous sublayer, $\delta_{0T}^+ \sim 10$ here according to [1].

By knowing the temperature profile T(y) on some section of the turbulent core, δ_{0T}^+ can be determined by a selection method by a methodology analogous to [11] on the basis of (15), where the value obtained will be unique. As an illustration (Fig. 2), the data of [12] were processed the concern the turbulent heat transfer for water flow in a plane channel ($Pr_w = 6.14$). Satisfaction of (15), or the identical equation (23) that is more convenient for this case, was confirmed upon assigning arbitrary values of δ_{0T}^+. The running values of $\chi(x, y)$ in (23) should remain constant in R_Θ [0.7; 1.0]. It is seen from Fig. 2 that upon giving $\delta_{0T}^+ = 9.2$, then $d\chi/dR < 0$ in the turbulent core, while $d\chi/dR > 0$ for $\delta_{0T}^+ = 7.5$ and only for $\delta_{0T}^+ = 8.0$ in R_Θ [0.7; 1.0] is $d\chi/dR = 0$, $\chi = \text{const}$, which corresponds to (23). Because of the absence of experimental values of ϑ_0^+ in [12] for $\delta_{0T}^+ = 7.5$ and 8.0, the Fourier law (17) was used in this example to determine ϑ_{0T}^+.

Represented in Fig. 3 in the form of the dependence $\Theta(R_\Theta)$ are all the temperature profiles [12] for water flow ($Pr_w = 6.14$-6.76) in a plane channel. In the turbulent core ($0.7 \le R_\Theta \le 1.0$) the presented data are described satisfactorily by the universal distribution (15). The closeness of the calculated values $\chi_{\delta T} = 0.41$-0.43 for $Pr > 1$ (Fig. 3) to the values $\tilde{\chi}_{\delta T} = 0.41$-$0.45$ for $Pr < 1$ (see Fig. 1) is in complete conformity with the analogy between the turbulent momentum transport and the heat (29).

Under complex flow conditions when one or several perturbing factors act on the stream, the universal profiles of the mean longitudinal velocity, the temperature, the concentration of substance, and the law of turbulent near-wall heat and mass transfer obtained can be utilized effectively by using the method elucidated in [6] since the relationships themselves do not change because of the absence of empirical constants therein, but the specifics of the perturbing action is taken into account in changes in the values of the criterial boundary functions $\Psi_{\delta i}$.

The results of this paper simplify the computation of the necessary characteristics for near-wall turbulent heat and mass transfer for different values of the molecular Prandtl and Schmidt numbers.

NOTATION

x, coordinate along the streamlined wall, m; y, coordinate along the normal to the wall, m; δ, δ_T, δ_c, respectively, the dynamic, thermal, and diffusion boundary layer thicknesses, m; δ_0, δ_{0T}, δ_{0C}, respectively, the dynamic, thermal, and diffusion viscous sublayer thicknesses; $\ell_* = \nu/u_*$, viscous scale, m; $y^+ = y/\ell_*$, dimensionless distance from the wall; u, mean longitudinal velocity, m/sec; $\vartheta = T_w - T$, mean temperature measured from the wall, K; $c = m_w - m$, mean relative mass concentration of passive impurity measured from the wall; ρ, density, kg/m^3; c_p, specific heat, J/(kg·K); ν, kinematic viscosity coefficient, m^2/sec; a, thermal diffusivity coefficient, m^2/sec; D, diffusion coefficient, m^2/sec; $Pr = \nu/a$, $Sc = \nu/D$, respectively, the molecular Prandtl and Schmidt numbers; Pr_t, Sc_t, turbulent Prandtl and Schmidt numbers, respectively; τ_w, tangential stress at the wall, N/m^2; q_w, heat flux density at the wall, W/m^2; j_w, mass flux density of the transportable passive impurity at the wall, kg/(m^2·sec); $u_* = \sqrt{\tau_w/\rho}$, dynamic velocity, m/sec; $\vartheta_* = q_w/\rho c_p u_*$, characteristic temperature, K; $c_* = j_w/\rho u_*$, characteristic relative mass concentration; $u^+ = u/u_*$, $\vartheta^+ = \vartheta/\vartheta_*$, $c^+ = c/c_*$, respectively, the dimensionless velocity, temperature, and concentration; $St = 1/u_\delta^+\vartheta_{\delta T}^+$, Stanton number; $St_c = 1/u_\delta^+ c_{\delta c}^+$, Stanton diffusion number; $\Psi = \ln(y/\delta_0)/(u^+ - u_0^+)$, $\chi = \ln[Pr(y - \delta_{0T})/\delta_{0T} + 1]/(\vartheta^+ - \vartheta_{0T}^+)$, $\Omega = \ln[Sc(y - \delta_{0c})/\delta_{0c} + 1]/(c^+ - c_{0c}^+)$, respectively, the dynamic, thermal, and diffusion criterial functions of the turbulent core; $\tilde{\Psi} = \ln y^+/(u^+ - 1)$, dynamic criterial function for zero pressure gradient; $\tilde{\chi} = \ln[Pr(y^+ - 1) + 1]/(\vartheta^+ - Pr)$, $\tilde{\Omega} = \ln[Sc(y^+ - 1) + 1]/(c^+ - Sc)$, thermal and diffusion criterial functions for $Pr \le 1$, $Sc \le 1$; $U = (u - u_0)/(u_\delta - u_0)$, $\Theta = (\vartheta - \vartheta_{0T})(\vartheta_{\delta T} - \vartheta_{0T})$, generalized dimensionless velocity and temperature; $R = \ln(y/\delta_0)/\ln(\delta/\delta_0)$, $R_\Theta = \ln[Pr(y - \delta_{0T})/\delta_{0T} + 1]/\ln[Pr(\delta_T - \delta_{0T})/\delta_{0T} + 1]$, generalized dimensionless coordinates in the dynamic and thermal boundary layers; $\tilde{U} = (u^+ - 1)/(u_\delta^+ - 1)$, generalized dimensionless velocity at zero pressure gradient; $\tilde{\Theta} = (\vartheta^+ - Pr)/(\vartheta_{\delta_T}^+ - Pr)$, generalized dimensionless temperature for $Pr \le 1$; $\tilde{R} = \ln y^+/\ln \delta^+$, $\tilde{R}_\Theta = \ln[Pr(y^+ - 1) + 1]/\ln[Pr(\delta_T^+ - 1) + 1]$, generalized dimensionless coordinates in the dynamic and thermal boundary layers. Subscripts δ, δ_T, δ_c, flow parameters for $y = \delta$, $y = \delta_T$, $y = \delta_c$, respectively; 0, OT, Oc, flow parameters, respectively, for $y = \delta_0$, $y = \delta_{0T}$, $y = \delta_{0c}$; t, turbulent core parameter; w, wall parameter; T, Θ, thermal layer parameters; c, diffusion layer parameter; i, transfer parameter of the i-th substance; i = 1, 2, 3, respectively, the momentum, heat, and substance transfer.

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